$$
\begin{gathered}
\text { Mathematics - Secondary } 3 \\
\text { Situational Problem - Kitchen Renovation }
\end{gathered}
$$

Your parents would like to renovate their kitchen this summer and have asked you to calculate for them the total cost for the job.

Your task is to calculate the total cost to renovate the kitchen; this includes the purchase and installation of new tiles and cabinet units, the cost of hardware, counter top and sink. You don't need to include taxes in your calculations.

## Part 1 - The Dimensions

First you need to find the actual dimensions of the kitchen which is a rectangular room; the working space is in a C shape. You know that the area of the kitchen is equal to the area of the living room. The plans of both rooms are given below with their dimensions, but they are not drawn to scale. All dimensions are in meters.


## Part 2 - Flooring

You need to purchase enough tiles to cover the entire kitchen area. The tiles will cost $\$ 45$ per $\mathrm{m}^{2}$.

As for the tile installation cost, it follows the rule: $T(a)=40 a+110$; where $T(a)$ is the tile installation cost, and a is the area of the kitchen in $\mathrm{m}^{2}$.

## Part 3 - Cabinet units and hardware

You realize that for the number of cabinet units that you need, their purchase cost is equal to their installation cost.

You will purchase the cabinet units from House Depot. Each cabinet unit costs $\$ 350$.
You will then have them installed by another store, Kitchens R Us. Their installation cost follows a linear function where the cost depends on the number of cabinet units needed as shown in the table below.

| Cabinet installation |  |
| :---: | :---: |
| Number of <br> Cabinets (n) | Cost (\$) <br> C(n) |
| 5 | 2170 |
| 10 | 3570 |

In addition, you will purchase from Kee Valley, the hardware (handles/knobs and hinges), which cost $\$ 10$ for each cabinet unit.

## Part 4 - Counter top

The counter top will be installed on the three walls of the kitchen, except where there are appliances, namely a stove and a fridge. To calculate the maximum space needed for the appliances, you need to solve the following inequality, where $p$ is the total length of appliances in meters.

$$
5(p+1) \geq 9 p-1.6
$$

The counter top is sold by the whole meter based on the total wall length required without the appliances length (p). It costs $\$ 92 / \mathrm{m}$. This price already includes the installation.

## Part 5 - The Sink

Your parents would like to install a corner sink. You have a choice between a double sink which costs $\$ 284$, and a single sink which costs $\$ 189$. They prefer a double sink but you are not sure if it will fit. So the kitchen designer at House Depot gives you the following top view drawing of the corner with sink and the necessary specifications.


Your task is to calculate the total cost to renovate the kitchen; this includes the purchase and installation of new tiles and cabinet units, the cost of hardware, counter top and sink. You don't need to include taxes in your calculations.

## Part 1 - The Dimensions

## Area of kitchen $=$ Area of living room

$x(x+2)=(\boldsymbol{x}+\mathbf{4})(\boldsymbol{x}-\mathbf{1})$
$x^{2}+2 x=x^{2}-x+4 x-4$
$x^{2}+2 x=x^{2}+3 x-4$
$2 x=3 x-4$
$\underline{x}=4$
Therefore the dimensions of the kitchen are 4 m by $\mathbf{6 m}$

## Part 2 - Flooring

$$
\begin{aligned}
\text { Area of kitchen } & =\text { Length } \times \text { width } \\
& =(6)(4) \\
& =\mathbf{2 4} \mathbf{~ m}^{2}
\end{aligned}
$$

| Cost of purchase $=45(24)$ |
| :--- | :--- |
| $=\$ 1080$ |$\quad$| Cost of installation: |
| :--- |
|  |
|  |

## Part 3 - Cabinets and hardware

Cost to purchase cabinets:

$$
C_{p}(n)=350 n
$$

Cost to install cabinets:

$$
\begin{aligned}
& a=\frac{3570-2170}{10-5}=\frac{1400}{5}=\$ 280 / \text { cabinet } \\
& b=y_{1}-a x_{1}=2170-(280)(5)=770
\end{aligned}
$$

Therefore the rule for installation: $\boldsymbol{C}_{\boldsymbol{i}}(\boldsymbol{n})=\mathbf{2 8 0 n}+\mathbf{7 7 0}$

## Solving the system of equations by comparison

| Cost of purchase $=$ cost of installation | Solving for costs: |
| :--- | :--- |
| $350 n=280 n+770$ | Purchase: |
| $\frac{-280 n}{70 n=770} \quad-280 n$ |  |
| $\frac{70}{70} \quad \frac{C_{p}(11)=350 n=350(11)=\$ \mathbf{3 8 5 0}}{70} \quad \therefore$ we need 11 cabinets |  |
| $\boldsymbol{n}=\mathbf{1 1} \quad$ | $C_{i}(11)=280 n+770=280(11)+770=\$ \mathbf{3 8 5 0}$ |

Cost of Hardware: 10(11) = \$ 110

## Part 4 - Counter top

| Maximum space needed for appliances: | Length of counter top : |
| :--- | :--- |
| $5(p+1) \geq 9 p-1.6$ | $4+6+4-1.65=12.35 \mathrm{~m}$ |
| $5 p+5 \geq 9 p-1.6$ |  |
| $5 p+5-5 \geq 9 p-1.6-5$ |  |
| $5 p \geq 9 p-6.6$ | Length of counter top to be purchased |
| $5 p-9 p \geq 9 p-9 p-6.6$ | (rounded to the nearest meter ) $=\mathbf{1 3} \mathbf{~ m}$ |
| $-4 p \geq-6.6$ |  |
| $\frac{-4 p}{-4} \leq \frac{-6.6}{-4}$ |  |
| $p \leq \mathbf{1 . 6 5}$ |  |

Cost of counter top : 92 (13) = \$ 1196

## Part 5 - The sink

| $\text { Length of diagonal } \begin{aligned} \overline{\boldsymbol{A C}} & =\sqrt{a^{2}+b^{2}} \\ & =\sqrt{65^{2}+65^{2}} \\ & =\sqrt{4225+4225} \\ & =\sqrt{8450} \\ & \approx \mathbf{9 1 . 9 2} \text { meters } \end{aligned}$ | $\text { OR } \begin{aligned} \overline{A G} & =91.92-49 \\ & =42.92 \mathrm{CM} \\ \overline{\boldsymbol{E F}}=\mathbf{2}(\overline{\boldsymbol{A G}}) & =\mathbf{2 ( 4 2 . 9})=\mathbf{8 5 . 8 4} \mathbf{~ c m} \\ & \leq \mathbf{9 2} \mathbf{~ c m} \end{aligned}$ |
| :---: | :---: |
| Since $\overline{A C}=\overline{B D}$, then $\overline{E F}<\overline{B D}<92 \mathrm{~cm}$ | Therefore the double sink does not fit |

## The single sink costs \$ 189

## Summary of expenses:

| Part | Cost (\$) |
| :---: | :---: |
| Tiles - Purchase | 1080 |
| Tiles - Installation | 1070 |
| Cabinets - Purchase | 3850 |
| Cabinets - Installation | 3850 |
| Cabinets - Hardware | 110 |
| Counter Top | 1196 |
| Sink | 189 |
| Total | $\mathbf{1 1 3 4 5}$ |

The Kitchen renovation will cost \$ 11345

